The preceding explanation of no form action theory has already laid the foundation for applying mathematics to philosophy. The relationship among the three no form actions already contains a certain mathematical structure. In this subsection, we will extract this mathematical structure and then conduct mathematical deductions to see what conclusions can be drawn. The way of studying philosophy in this section is completely different from the preceding ones. This section aims to use the method of rigorous mathematical operations to obtain the relationships among concepts, and to establish concepts upon a mathematical structure. This makes philosophical research, like scientific research, possess systematicity, rigor, and precision, and allows philosophy to attain the status it rightfully deserves. This can solve the long-standing criticisms of philosophy—that it is too vague, subjective, and lacks obvious progress.

The traditional philosophical research preceding this used only imprecise speculation and vague intuition (although they are indispensable and their role is undeniable, they are also incomplete), and the philosophical viewpoints obtained were, for the most part, unconvincing. Now, these shortcomings in philosophical research will become a thing of the past. As no form action theory is successfully constructed upon rigorous mathematics, philosophical research will be elevated to a new level, philosophy will enter a new era, and philosophy, as an ancient field of study, will become a mature discipline.

Since the content of this subsection primarily makes use of group theory from mathematics, for readers unfamiliar with groups, I will first give a brief introduction to the group.

Let G be a non-empty set, and let a binary operation '·' be defined on G that satisfies the following conditions:

1) Closure: For any a, b in G, there exists a unique element c in G such that $a \cdot b = c$.

2) Associativity: For any a, b, c in G, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

3) Identity Element: There exists an element e in G such that for any a in G, $e \cdot a = a \cdot e = a$.

4) Inverse Element: For any a in G, there exists an element b in G such that $a \cdot b = b \cdot a = e$. 'a' is said to be invertible, and 'b' is called the inverse of 'a', denoted as a^{-1}.

Then G is said to form a group.

(1) The Klein four-group: No Form V

The Klein four-group (Klein, Lectures on the Icosahedron, 1884) provides a structure that can represent the three actions of no form action theory and their mutual transformation. The Klein four-group is chosen because it captures the core idea of cyclical transformation among different elements, which aligns very well with the concept of the mutual transformation of the three actions.

(1.1) The Klein four-group structure:

The Klein four-group, usually denoted as V or K_4, has four elements {e, a, b, c} and the following operation table (the binary operation here is '+'):

+	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

(1.2) Key properties of the Klein four-group:

Each element is its own inverse: a + a = e, b + b = e, c + c = e.

The operation is commutative: a + b = b + a = c, etc.

Mapping the no form actions to the group elements:

e: Represents no form, emphasizing that no form has not yet combined with form, and thus does not generate any no form action. It is taken as the identity element.

a: Represents manifestation action.

b: Represents motive force action.

c: Represents isolation action.

(1.3) Explaining the Group Operation:

The group operation (+) represents the transformation of one no form action into a third no form action through another no form action.

1) x + e = x: No form e, as the identity element, does not change any action.

2) c + c = e: According to no form united transformation, isolation action requires another no form action to be transformed into a third no form action. Therefore, isolation action cannot change on its own, which in fact means that nothing happens. Isolation action acting on itself in fact generates no action; it belongs to pure no form. Therefore, the result of c + c is e.

3) b + b = e: By the same logic, the result of b + b is e.

4) a + a = e: By the same logic, the result of a + a is e.

5) a + b = c: Represents that the transformation of manifestation action into isolation action requires motive force action.

6) a + c = b: Represents that the transformation of manifestation action into motive force action requires isolation action.

7) b + c = a: Represents that the transformation of motive force action into manifestation action

requires isolation action.

8) b + a = c: Represents that the transformation of motive force action into isolation action requires manifestation action.

9) c + a = b: Represents that the transformation of isolation action into motive force action requires manifestation action.

10) c + b = a: Represents that the transformation of isolation action into manifestation action requires motive force action.

For some things, the transformation among manifestation action, motive force action, and isolation action is commutative (e.g., a + b = b + a = c), and can thus be represented by this group. For example, the three fundamental laws of formal logic are like this; the transformation of any two laws into the third does not require consideration of order. It is clear that if A (manifestation action), B (motive force action), and C (isolation action) constitute a no form integrated transformation, then it can be represented by this group. Conversely, if A (manifestation action), B (motive force action), and C (isolation action) constitute such a group, then they constitute a no form integrated transformation. This shows that this group is meaningful; it represents a no form integrated transformation.

Thus, we call this group No Form V. No Form V elevates no form integrated transformation from a philosophical concept to the mathematical level, making it more precise and rigorous. No Form V is the abstract mathematical expression of no form integrated transformation. It can not only represent a no form integrated transformation, but it can also verify whether a certain transformation meets the requirements of an integrated transformation, and vice versa. No form integrated transformation is no longer merely a philosophical idea, but an operable and verifiable mathematical model. This means that one can actively operate on philosophical concepts, rather than just abstractly describing them. This gives no form action theory a higher level of theoretical rigor.

(1.4) No Form V and the Cyclic Group Z_2

However, for this No Form V that I have constructed, what is more important is that the Klein four-group is isomorphic to $Z_2 \times Z_2$. The cyclic group Z_2 (or C_2) = {0, 1}, and $Z_2 \times Z_2 = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. The operation in $Z_2 \times Z_2$ is component-wise addition modulo 2: $(x_1, y_1) + (x_2, y_2) = ((x_1 + x_2) \bmod 2, (y_1 + y_2) \bmod 2)$. If we use 0 to represent no form (note that the 0 here emphasizes its relationship with form) and 1 to represent form, then Z_2 can be seen as the cyclic group formed by the combination of no form and form.

The operation in the cyclic group Z_2 is addition modulo 2: {0, 1}, with the identity element being 0. 0 + 0 = 0 (mod 2), 0 + 1 = 1 (mod 2), 1 + 0 = 1 (mod 2), 1 + 1 = 0 (mod 2). 1 + 1 = 0 (mod 2) can be explained as: form cannot be developed from itself. Form can only be developed through no form (for example, 0 + 1 = 1 (mod 2)); it can only be developed by returning to no form. Therefore, 1 + 1 = 0. Conversely, no form also needs form to function. In other words, this embodies their indivisibility. Therefore, Z_2 represents the relationship between no form and form.

Since Z_2 possesses the relationship of form and no form, No Form V can be seen as arising from the combinatorial changes of the direct product $Z_2 \times Z_2$. Then, (0, 0) can be seen as the identity element, no form e (although no form changing from 0 to e is still no form, its status has changed: from a relationship with form to a relationship with the three no form actions). (0, 1) can be seen as manifestation action a; (1, 0) can be seen as motive force action b; and (1, 1) can be seen as isolation action c.

In other words, the elements 0 (representing no form) and 1 (representing form) from Z_2 are combined in No Form V into the three concrete no form actions. The combinatorial process from (0, 0)... to (1, 1) shows a process of gradually strengthening form: (0, 0) represents the absence of any no form action; (0, 1) represents manifestation action a, in which no form is dominant and form is subordinate; (1, 0) represents motive force action b, in which form is dominant and no form is subordinate; and (1, 1) represents isolation action c, in which form is completely dominant. This process can be seen as a dynamic display of the interaction and gradual transformation between no form and form. At the same time, it reveals the generative order of the different actions in no form theory and their interrelations. No Form V can be seen as an extension of Z_2. In it, the relationship between form and no form is not merely a logical complementary relationship, but also achieves a dynamic transformation and balance through manifestation action, motive force action, and isolation action.

Since the only form of a group with two elements is Z_2, and the product of two Z_2 groups can only be the direct product, the group that is extended from Z_2 itself, as having the relationship of form and no form, can only be No Form V (in the sense of isomorphism). This is to say that from a mathematical perspective, we have also determined that there are only three of the most fundamental actions: manifestation, motive force, and isolation. And these three actions are by no means arbitrary.

In fact, the process of creating no form action theory began with the two-dimensional theory composed of form and no form, and then came the three actions of no form. This is the same as the order of extending from Z_2 to No Form V. However, the process of creating no form action theory was carried out only through the method of intuitive analysis and did not clarify the relationship between the two-dimensional theory and the three actions of no form. But through mathematical means, their relationship has been clearly obtained: just by combining form and no form in different ways, the three actions of no form can be obtained. In fact, it would be very difficult to think of this using only the method of intuitive analysis. But this also shows that the no form action theory created through intuition is not arbitrary, but was implicitly guided by an underlying logical and mathematical structure.

(2) The Direct Product: No Form V × V

We can also form a direct product group of (No Form V) × (No Form V), which we will call No Form V × V. Let No Form V_1 = {e, a, b, c} and No Form V_2 = {e, a, b, c}. Then, No Form V × V = {(e, e), (e, a), (e, b), (e, c), (a, e), (a, a), (a, b), (a, c), (b, e), (b, a), (b, b), (b, c), (c, e), (c, a), (c, b), (c, c)}. In this way, No Form V × V has 16 elements and has two dimensions: V_1 and V_2.

Since No Form V × V is a group, its elements can be operated upon: (x, y) + (m, n) = (x + m, y + n). For example, (a, c) + (e, b) = (a + e, c + b) = (a, a). That is to say, the operation between them is an operation within their respective dimensions; the operations of the two dimensions are independent of each other. This direct product in fact transforms the one-dimensional no form united transformation into a two-dimensional no form united transformation. No Form V × V has 16 elements, so there are a total of 16 × 16 = 256 operations among these 16 elements. Since it is a group, these 256 operations still have 16 results.

The product between groups is also an operation, but it is, in fact, already different from the operation between elements within a single group. The operation between groups has become of the form (x, y), where x belongs to No Form V_1 and y belongs to No Form V_2. What is the philosophical meaning of this?

Viewing one no form action (y) from the perspective of another no form action (x) means 'the x in y'. For example, viewing isolation action from the perspective of motive force action is independence, which means 'the motive force action in isolation action'. If we regard this as an operation, for example, viewing isolation action (c) from the perspective of motive force action (b) is: (c, b). We might as well call this operation 'perspective operation'. Thus, the operation of the product between groups is 'perspective operation'. This operation breaks through the limitation of a single dimension and brings about an unfold-manifestation of the multidimensional connections among the no form actions. 'Perspective operation' emphasizes the importance of perspective, that is, one action may have different meanings under different perspectives. For example, 'independence' and 'generation' are two different manifestations under mutual perspectives.

The concepts represented by the 16 elements of No Form V × V are, respectively:

no form (e, e), the self (a, a), being-for-itself (b, b), self-limitation (c, c), transparency (e, a), freedom (e, b), being (e, c), manifestation (a, e), motive force (b, e), isolation (c, e), Immediacy (a, b), identity (a, c), generation (b, c), change (b, a), independence (c, b), and distinction (c, a).

Since these 256 operations are rather numerous, we will classify and arrange the 16 elements of No Form V × V to study the relationships among them. This is shown in the table below:

no form (e, e)	manifestation (a, e)	motive force (b, e)	isolation (c, e)
the self(a, a)	transparency (e, a)	freedom (e, b)	being (e, c)
being-for-itself (b, b)	change (b, a)	generation (b, c)	identity (a, c)
self-limitation (c, c)	immediacy (a, b)	independence (c, b)	distinction (c, a)

In this way, the perspective operation and the group operation of No Form V can be combined; that is, viewing a problem from different perspectives and no form united transformation are fused together. This allows complex philosophical problems to be expressed within a unified framework. No Form V is the basic structure of the three no form actions, while No Form V × V is a higher-dimensional extension of this structure.

The elements in No Form V × V can undergo group operations. For example, identity (a, c) + independence (c, b) = change (b, a). This means that identity and independence can be transformed into change, or that for identity to be transformed into change, independence is required. Furthermore, it can be verified that these three constitute a no form integrated transformation; that is, they can be mutually transformed among themselves. This, in fact, transforms a one-dimensional no form integrated transformation into a two-dimensional no form integrated transformation.

In this way, we can obtain the relationships among these concepts through the operations of the group. This method transcends mere description or intuitive assertion; that is, the relationships among concepts can be independently verified through mathematical operations. Compared to the one-dimensional no form integrated transformation, the two-dimensional structure allows for the embedding of the interactions among more concepts into a more complex mathematical framework, making the analysis deeper and more systematic. This is a way of obtaining the relationships among concepts through mathematical operations. In this manner, we can discover relationships among concepts that were previously undiscovered. This two-dimensional group structure allows for the verification of whether three concepts satisfy the conditions of a no form integrated transformation, that is, whether the transformation of each concept into another requires the third concept.

The introduction of group operations elevates no form action theory from a purely philosophical theory to a mathematical logical framework that can be operated upon and used for deduction. It is not only a descriptive tool, but also a tool for analysis and inference. This is a method that utilizes the power of mathematical operations to analyze and understand complex philosophical problems in a way that was previously impossible.

All of the two-dimensional no form integrated transformations in No Form V × V can be obtained. There are a total of the following 35 sets, and the three concepts in each set are mutually transformable:

1) {transparency (e,a), freedom (e,b), being (e,c)}

2) {transparency (e,a), manifestation (a,e), the self (a,a)}

3) {transparency (e,a), Immediacy (a,b), identity (a,c)}

4) {transparency (e,a), motive force (b,e), change (b,a)}

5) {transparency (e,a), being-for-itself (b,b), generation (b,c)}

6) {transparency (e,a), isolation (c,e), distinction (c,a)}

7) {transparency (e,a), independence (c,b), self-limitation (c,c)}

8) {freedom (e,b), manifestation (a,e), Immediacy (a,b)}

9) {freedom (e,b), the self (a,a), identity (a,c)}

10) {freedom (e,b), motive force (b,e), being-for-itself (b,b)}

11) {freedom (e,b), change (b,a), generation (b,c)}

12) {freedom (e,b), isolation (c,e), independence (c,b)}

13) {freedom (e,b), distinction (c,a), self-limitation (c,c)}

14) {being (e,c), manifestation (a,e), identity (a,c)}

15) {being (e,c), the self (a,a), Immediacy (a,b)}

16) {being (e,c), motive force (b,e), generation (b,c)}

17) {being (e,c), change (b,a), being-for-itself (b,b)}

18) {being (e,c), isolation (c,e), self-limitation (c,c)}

19) {being (e,c), distinction (c,a), independence (c,b)}

20) {manifestation (a,e), motive force (b,e), isolation (c,e)}

21) {manifestation (a,e), change (b,a), distinction (c,a)}

22) {manifestation (a,e), being-for-itself (b,b), independence (c,b)}

23) {manifestation (a,e), generation (b,c), self-limitation (c,c)}

24) {the self (a,a), motive force (b,e), distinction (c,a)}

25) {the self (a,a), change (b,a), isolation (c,e)}

26) {the self (a,a), being-for-itself (b,b), self-limitation (c,c)}

27) {the self (a,a), generation (b,c), independence (c,b)}

28) {Immediacy (a,b), motive force (b,e), independence (c,b)}

29) {Immediacy (a,b), change (b,a), self-limitation (c,c)}

30) {Immediacy (a,b), being-for-itself (b,b), isolation (c,e)}

31) {Immediacy (a,b), generation (b,c), distinction (c,a)}

32) {identity (a,c), motive force (b,e), self-limitation (c,c)}

33) {identity (a,c), change (b,a), independence (c,b)}

34) {identity (a,c), being-for-itself (b,b), distinction (c,a)}

35) {identity (a,c), generation (b,c), isolation (c,e)}

Furthermore, each of these triplets, together with the identity element (e, e), forms a subgroup of order 4 of No Form V × V, and they are all Klein four-groups. Since No Form V is a Klein four-group, and No Form V × V, as an extension of No Form V, in turn exhibits the 'Klein four-group' as a subgroup. This subtle recursion will be very useful.

Additionally, there are 16 other two-dimensional no form integrated transformations of the form {no form (e, e), (x, y), (x, y)}. In fact, they are the subgroups of order 2 of No Form V × V.

No Form V × V has 15 subgroups of order 8, all of which are isomorphic to the direct product $Z_2 \times Z_2 \times Z_2$. Each subgroup is a closed system.

H_1 = {no form (e,e), the self (a,a), Immediacy (a,b), identity (a,c), manifestation (a,e), transparency (e,a), freedom (e,b), being (e,c)}

H_2 = {change (b,a), no form (e,e), being-for-itself (b,b), generation (b,c), motive force (b,e), transparency (e,a), freedom (e,b), being (e,c)}

H_3 = {change (b,a), no form (e,e), the self (a,a), isolation (c,e), motive force (b,e), manifestation (a,e), transparency (e,a), distinction (c,a)}

H_4 = {no form (e,e), being-for-itself (b,b), Immediacy (a,b), generation (b,c), identity (a,c), isolation (c,e), transparency (e,a), distinction (c,a)}

H_5 = {independence (c,b), no form (e,e), being-for-itself (b,b), Immediacy (a,b), isolation (c,e), motive force (b,e), manifestation (a,e), freedom (e,b)}

H_6 = {change (b,a), independence (c,b), no form (e,e), the self (a,a), generation (b,c), identity (a,c), isolation (c,e), freedom (e,b)}

H_7 = {change (b,a), independence (c,b), no form (e,e), being-for-itself (b,b), identity (a,c), manifestation (a,e), being (e,c), distinction (c,a)}

H_8 = {independence (c,b), no form (e,e), the self (a,a), Immediacy (a,b), generation (b,c), motive force (b,e), being (e,c), distinction (c,a)}

H_9 = {change (b,a), no form (e,e), the self (a,a), being-for-itself (b,b), self-limitation (c,c), Immediacy (a,b), isolation (c,e), being (e,c)}

H_{10} = {no form (e,e), self-limitation (c,c), generation (b,c), identity (a,c), isolation (c,e), motive force (b,e), manifestation (a,e), being (e,c)}

H_{11} = {no form (e,e), the self (a,a), being-for-itself (b,b), self-limitation (c,c), identity (a,c), motive force (b,e), freedom (e,b), distinction (c,a)}

H_{12} = {change (b,a), no form (e,e), self-limitation (c,c), Immediacy (a,b), generation (b,c), manifestation (a,e), freedom (e,b), distinction (c,a)}

H_{13} = {change (b,a), independence (c,b), no form (e,e), self-limitation (c,c), Immediacy (a,b), identity (a,c), motive force (b,e), transparency (e,a)}

H_{14} = {independence (c,b), no form (e,e), the self (a,a), being-for-itself (b,b), self-limitation (c,c), generation (b,c), manifestation (a,e), transparency (e,a)}

H_{15} = {independence (c,b), no form (e,e), self-limitation (c,c), isolation (c,e), transparency (e,a), freedom (e,b), being (e,c), distinction (c,a)}

In No Form V × V, we find many oppositions. Among them, generation (b,c) and independence (c,b) are in opposition (it can be seen from the direction of the two elements (b,c) and (c,b) that they are in opposition); they are unified in cause. Change (b,a) and Immediacy (a,b) are in opposition; they are unified in opening. Identity (a,c) and distinction (c,a) are in opposition; they are unified in ground. We have already seen these three pairs of opposites in the subsection "Dialectical Logic". However, for now, these oppositions and unities are still obtained through the method of conceptual analysis, not through mathematical operations. Later, mathematical operations will be used to obtain these relationships.

We also see that: manifestation (a,e) and transparency (e,a), motive force (b,e) and freedom (e,b), and isolation (c,e) and being (e,c) are all in opposition. Note, the two-dimensional expression of these six concepts is an expression in the manner of a limit. For instance, being (e,c) is the limit expression of 'being'. The meaning of being (e,c) is viewing no form from the perspective of form and reaching being in the manner of a limit. They are respectively the limits of the following six concepts: essence and opening, subject and cause, substance and ground (as has been discussed in the subsection "viewing no form from the perspective of form"). This mode of expression highlights the role of the limit.

Essence is not open; that which is open is the phenomenon. Therefore, they are in opposition. Similarly, since it is the subject, there should be no cause; therefore, they are in opposition. Similarly, since it is an independently existing substance, there should be no ground; therefore, they are in opposition. Through their opposition, the opposition of their limits can also be obtained. For example, substance and ground are in opposition. Their limits are isolation (c,e) and being (e,c) respectively, and these two limits are also in opposition. Because isolation (c,e) can also be seen as a substance taken to its limit, and being (e,c) can also be seen as a ground taken to its limit (since viewing no form directly from form in the mode of isolation is 'ground', therefore, the limit of continuously reducing the form of ground is being (e,c); being (e,c) is its own ground. The relationship between isolation (c,e) and substance is similar).

Ground and the concrete are concepts of the same category, but a distinction arises due to their relativity. For example, take three concepts a, b, and c. If c is the ground of b, then b is the concrete of c. But b is also the ground of a. Then b can be both concrete and ground; a difference arises only due to relativity. The direction of this relativity is opposite, so ground and the concrete are in opposition. By the same logic, cause and result are in opposition. By the same logic, opening and concealment are in opposition.

(3) The Extension Field of No Form V

Taking isolation action 'c' as the identity element, we can extend this No Form V into the field F_4. This is shown in the table below:

·	e	c	a	b
e	e	e	e	e
c	e	c	a	b
a	e	a	b	c

b	e	b	c	a

No Form V = {e, a, b, c} is extended into the field F_4: e is the zero element, c is the identity element, and the multiplication '·' is the second operation.

We see that the three no form actions have been successfully constructed on the Klein four-group, and this group has been extended into the field F_4. We might as well call this field 'VFc' (where F represents a field, and c represents that the identity element of this field is c). Thus, the elements in VFc = {e, a, b, c} can undergo both the '+' operation and the '·' operation.

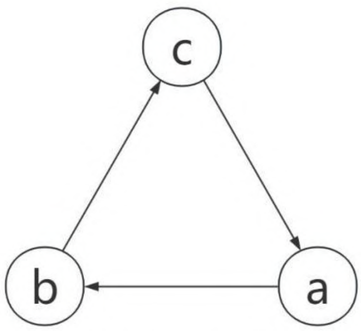
(3.1) The Cyclic Group C_3

VFc, as the extension field of No Form V = {e, a, b, c}, forms a group C_3 = {c, a, b}. This group is a cyclic group, with the identity element being 'c'. The operations within it are:

c · c = c, c · a = a, c · b = b

a · c = a, a · a = b, a · b = c

b · c = b, b · a = c, b · b = a



Cayley graph of C_3

Due to the necessary reliability of mathematics, this C_3 must certainly have its practical value and meaning. I explain this cyclic group in this way: it can be seen as a pure world of isolation composed of isolation action. 'c' represents an isolated thing, 'a' represents affirmation, and 'b' represents negation. We see that: c · a = a, a · a = b, b · a = c. Starting from 'c', then to 'a', and then to 'b', successively multiplying by 'a' forms a cycle (please see the Cayley graph of C_3).

1) c · a = a represents the affirmation of the isolated thing 'c'.

2) a · a = b: 'a' represents the affirmation of the affirmed 'c'. Since 'c' has already been affirmed, how can it be affirmed again? It is by affirming non-c to re-affirm the affirmed 'c'; that is, it is the negation of 'c'.

3) b · a = c: 'b' represents the affirmation of non-c. Since non-c has already been affirmed, to re-affirm the affirmed non-c is to affirm 'c'.

These three steps complete a cycle. This cycle expresses the characteristics of isolation action: independence and distinction. As has been discussed in the subsection "Dialectical Logic," independence is to affirm oneself and negate the other (non-self). 'c' and 'non-c' in this cycle express the characteristic of distinction. This cycle more clearly and completely expresses these two concepts of independence and distinction, which also completely expresses the concept of isolation.

From $a \cdot a = b$, we see that affirmation multiplied by affirmation equals negation. This is different from the 'affirmation of an affirmation equals affirmation' that we usually associate with formal logic. Note, the logic here is not yet formal logic. In fact, the cyclical expression of C_3 is the foundation for both formal logic and dialectical logic. Because this cyclical expression of C_3 is expressing the two concepts of independence and distinction, and these two concepts are the foundation for formal logic and dialectical logic (as will be seen later).

With the explanation of these three steps, the other operations can be explained:

4) $a \cdot b = a \cdot (a \cdot a) = b \cdot a = c$. Here, $a \cdot b$ is transformed into $b \cdot a$.

5) $b \cdot b = b \cdot (a \cdot a) = (b \cdot a) \cdot a = c \cdot a = a$. Here, $b \cdot b = a$ expresses that negation multiplied by negation equals affirmation.

6) $a \cdot c = a \cdot (a \cdot b) = (a \cdot a) \cdot b = b \cdot b = a$. Similarly, $b \cdot c = b$.

7) $c \cdot c = (a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b = c$.

8) $x \cdot e = e$ (or $e \cdot x = e$)

This formula shows that the multiplicative operation of any no form action and the no form action 'e' is the no form action 'e'. This formula can be written as $x \cdot e = x \cdot (x + x)$. For example, $b \cdot e = b \cdot (b + b) = b \cdot b + b \cdot b = a + a = e$. From this perspective, it can be understood why the result of this formula is 'e'. This 'e' is the identity element 'e' in No Form V. When No Form V is extended into the field VFc, 'e' participates in the multiplicative operation of VFc ($x \cdot e = e$) and becomes 'nothingness' in the world of isolation. From the perspective of the world of isolation, this 'e' is something completely devoid of anything. At this point, this 'nothingness' expresses the fact that a concept has no attributes, or does not have a certain attribute. For example, a line segment has no area. This 'nothingness' is a transformed no form; it is also a 'no form' simulated in the world of isolation. The meaning of $x \cdot e = e$ is: all forms, upon contact with 'nothingness', are reduced to a state where attributes are completely absent.

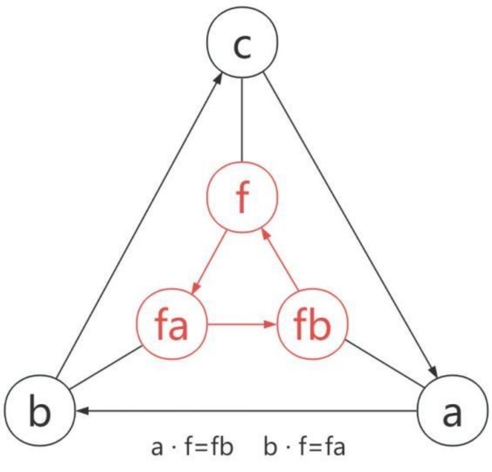
Let us now further clarify affirmation and negation. 'Is' itself is manifestation, but it is associated with the motive force action 'affirmation'. Similarly, 'is not' itself is also manifestation, but it is also associated with the motive force action 'negation'. When we want to affirm a concept, we are using the manifested 'is' to transform it into an isolated concept; this is a no form united transformation. For example, 'a is b'.

In fact, affirmation and 'is' are the prerequisites for an isolated concept; affirmation and an isolated concept are the prerequisites for 'is'; and 'is' and an isolated concept are the prerequisites for affirmation. They constitute a no form integrated transformation. The same is true for negation and 'is not'. 'Is' and 'is not' are in opposition, and affirmation and negation are also in opposition. 'Is' can be transformed into 'is not' through negation. 'Is' can also be transformed into affirmation through an 'isolated concept'. These are different no form united transformations. We see that affirmation and negation are always bound together; they are indivisible. They interact with each other and are mutually transformed. This indivisibility is precisely their mathematical embodiment as a closed system in the group C_3.

In Z_2, there is only the basic operation of symmetry (such as the simple inversion of 0 to 1). No Form V extends the operation by introducing the three no form actions (manifestation, motive force, isolation). The field VFc can be seen as a theoretical connection point from No Form V to C_3. And C_3 further provides a mathematical structure that can express isolation action (independence and distinction), making it no longer merely a philosophical speculation, but a concept with a mathematical structure.

(3.2) The Dihedral Group D_3

Since C_3 is a cyclic group, it can be extended into the dihedral group $D_3 = \{1, a, a^2, f, fa, fa^2\}$, where f is the reflection of C_3. Then, the set $FA = \{f, fa, fa^2\}$ is the mirror image of C_3; that is, FA and C_3 are in opposite directions. Corresponding to the elements of C_3, we can write D_3 as: $D_3 = \{c, a, b, f, fa, fb\}$ (these two notations are isomorphic), where c is the identity element, and the mirror image of C_3 is $FA = \{f, fa, fb\}$.



Cayley graph of D_3

Then, for no form action theory, we can take C_3 as A, and the mirror image FA as non-A. Thus, since $a \cdot f = fb$, the action of f on a is to turn the affirmation of A into the negation of non-A. Similarly, $b \cdot f = fa$ is to turn the negation of A into the affirmation of non-A. Conversely, $fa \cdot f = b$; the action of f on fa is to turn the affirmation of non-A into the negation of A. Similarly, $fb \cdot f = a$ is to turn the negation of non-A into the affirmation of A. In other words, the affirmation of A corresponds to the negation of non-A; the negation of A corresponds to the affirmation of non-A.

In fact, 'a' (the affirmation of A) and 'fb' (the negation of non-A) are not in opposition, but are complementary, because they both point towards A. Whereas 'a' (the affirmation of A) and 'fa' (the affirmation of non-A) are in opposition. The action of f on the entire C_3 is what turns A into non-A.

The action of the reflection f is negation (motive force action). 'f' enables the mutual transformation between A (C_3) and non-A (FA), and unifies them into D_3. This is, in fact, dialectical logic; it is the precise mathematical expression of the dialectical relationship between A and non-A. C_3 specifies the affirmation and negation that serve as the foundation of logic, and when they are applied to themselves (e.g., $a \cdot f = fb$), the mathematical expression of dialectical logic is generated.

A, as a thing of isolation (isolation action), through the negation of A (motive force action), obtains the reverse manifestation of non-A (manifestation action). This is a no form united transformation. In this process, the three no form actions are also generated at the same time. They are also mutually isolated. Therefore, A and non-A are also mutually isolated and different from each other. A and non-A thus acquire distinguishability. The generation of this distinguishability is due to the unification of A and non-A in D_3 (D_3 is the standard for their distinction); they are mutually isolated parts of D_3. This embodies the characteristic of distinction of this isolation action.

In D_3, due to the action of 'f', the relationship between A and non-A appears. This is a mathematical evolutionary extension, whereas in C_3, one remains only at the level of operations within A. We see that what is expressed in C_3 is the relationship between affirmation and negation; whereas what is expressed in D_3 is the relationship between A and non-A. Their relationship is expressed through affirmation and negation. The analysis of C_3 and D_3 shows that the relationship between A and non-A is associated with the relationship between affirmation and negation; they are bound together. Does this express the rules of formal logic?

Note, here there is only the relationship between A and non-A, not yet between A and B. Therefore, according to the viewpoint of no form action theory, to affirm A is 'A is A'. 'A is not non-A' is fb. So, the action of f on a ($a \cdot f = fb$) is to turn 'A is A' into 'A is not non-A'. The action of f on fb ($fb \cdot f = a$) is to turn 'A is not non-A' into 'A is A'. Then, the law of non-contradiction is: 'A is not non-A'. And the law of the excluded middle is: 'if A is A, then it is not non-A; if A is not non-A, then it is A'. And the law of identity is: 'A is A'. Note, 'A is A' (affirming oneself) and 'A is not non-A' (negating the other) are complementary expressions. 'A is A' and 'A is non-A' are the ones in opposition.

We call the three fundamental laws deduced from D_3: the D_3 version of the three fundamental laws of formal logic.

Why are they expressed as 'A is A' and 'A is not non-A'? These two expressions are precisely the manifestation of the two opposing aspects of the characteristic 'independence' of isolation: affirming oneself and negating the other. And the dihedral group D_3 happens to fit this expression. Of course, one can also use 'non-A is non-A' as the basis to evolve a non-A version of the three fundamental laws from the perspective of non-A. The laws of these two forms of logic are symmetrical.

In this way, it can be easily seen that the three fundamental laws of the D_3 version constitute a no form integrated transformation:

1) The law of identity ('A is A') and the law of non-contradiction ('A is not non-A') together make up the law of the excluded middle: 'if A is A, then it is not non-A; if A is not non-A, then it is A'.

2) The law of identity ('A is A') and the law of the excluded middle together can yield the law of non-contradiction: from 'A is A' and 'if A is A, then it is not non-A', one can obtain 'A is not non-A'.

3) Similarly, the law of non-contradiction ('A is not non-A') and the law of the excluded middle together can yield the law of identity: from 'A is not non-A' and 'if A is not non-A, then it is A', one can obtain 'A is A'.

We can define truth and falsity: let us call 'A is A' true, and 'A is not A' false. Thus, true and false become mutually exclusive.

Since true and false are mutually exclusive, if we substitute A = true and non-A = false into the three fundamental laws of the D_3 version, we can then obtain the truth-falsity version of the three fundamental laws.

Law of Identity: true is true.

Law of Non-Contradiction: true is not false.

Law of the Excluded Middle: if true is true, then it is not false; if true is not false, then it is true.

Due to symmetry, we can also substitute A = false and non-A = true into the three fundamental laws of the D_3 version, which would then yield the falsity-truth version of the three fundamental laws.

Law of Identity: false is false.

Law of Non-Contradiction: false is not true.

Law of the Excluded Middle: if false is false, then it is not true; if false is not true, then it is false.

For the expression 'A is B', if A is indeed in set B, it can be written as an extension of 'A is A' (as discussed in the subsection "Formal Logic"): '"A is the A in B" = true'. This is abbreviated as '"A is B" = true'. Conversely, if A is not in set B, then it can be written as an extension of 'A is not A': '"A is not the A in non-B" = false'. This is abbreviated as '"A is B" = false'. The meaning of the extended form of 'A is A' is that we are not concerned with the specific way in which A is itself, but only with whether A is itself or not.

Thus, for the expression 'A is B', regardless of what its state is, if we jointly use the truth-falsity version and the falsity-truth version of the three fundamental laws, we can obtain:

'"A is B" is either not false or is true, and is either not true or is false'. This can be simplified to: '"A is B" is either true or false'. This is the expression of the law of the excluded middle in traditional formal logic.

'"A is B" being true means it is not false, and being false means it is not true'. This can be simplified to: '"A is B" cannot be both true and false'. This is the expression of the law of non-contradiction in traditional formal logic.

'"A is A"' is a universal expression.

These are the three fundamental laws of traditional formal logic. Early traditional formal logic was: 'A is A'; 'it cannot both be and not be' (this is the law of non-contradiction); 'it is either a thing or it is not a thing' (this is the law of the excluded middle). This is the 'is' version of the laws of formal logic. Since true and false express a proposition (e.g., A is B), the three fundamental laws of traditional formal logic are an equivalent expression of the three fundamental laws of early traditional formal logic.

Because the three fundamental laws of traditional formal logic are an extension of the three fundamental laws of the D_3 version, then based on the no form integrated transformation constituted by the three fundamental laws of the D_3 version, it can be fully inferred that the three fundamental laws of traditional formal logic also constitute a no form integrated transformation.

In this way, using no form action theory, using a mathematical approach, and then through the transitional evolution of the 'truth-falsity' version and the 'falsity-truth' version of the three fundamental laws, we have evolved the three fundamental laws of traditional formal logic. Note, we are not proving the three fundamental laws of formal logic, but rather evolving the fundamental laws of formal logic as we commonly know them from the laws contained in the most basic Z_2, which represents form and no form. The world of isolation C_3 comes from the extension field of No Form V, and No Form V in turn comes from the extension of Z_2. In other words, the three fundamental laws of formal logic are ultimately founded on the basis of form and no form. This evolution is from the ontological form and no form gradually evolving to the methodological formal logic. Of course, this evolution is completed through epistemology (e.g., the knowledge of the two characteristics of isolation action, the knowledge of affirmation and negation, the knowledge of groups, etc.). This provides a deeper ontological foundation for the three fundamental laws of formal logic and avoids the problem of infinite regress that arises when attempting to base logic on other 'logical' but equally untestable assertions.

Although the rules of formal logic have already been used in epistemology, such an evolution is, in fact, a no form united transformation (in the previous subsection "The Relationship of Motive Force: The Mutual Transformation Among the Three No Form Actions," it was already clarified that ontology, epistemology, and methodology constitute a no form integrated transformation). This can be seen as the evolution of the rules in ontology into the rules of formal logic. Therefore, this evolution cannot be seen as a circular argument. This shows that the three fundamental laws of formal logic are not arbitrary or conventional, but originate from the underlying structure of the universe. These fundamental laws are necessary and unavoidable. This has been shown step-by-step through a structured method, and this process of demonstration has allowed us to clearly see this necessity.

The D_3 version, the 'truth-falsity' (or 'falsity-truth') version, and the traditional version of the three fundamental laws of formal logic that we have evolved are each more concrete than the last. They are evolved with the 'is' version of the laws of formal logic as a prerequisite, because 'is' (affirmation) and 'is not' (negation) have already appeared in C_3. Another important aspect is that in this process of evolution, we have obtained the relationship between the identity of 'is' ('A is A'), truth and falsity, and 'A is B'.

Thus, we can clearly see that in D_3, not only is dialectical logic expressed, but also formal logic. They are bound together in a complementary relationship. This maps the two complex and abstract logics onto a rigorous and precise mathematical structure. This mathematical model can also ensure logical consistency through the rigor of the mathematical structure. This transcends analogy and shows that the structural laws of thought are in fact encoded in the mathematical group. And the operation of C_3 provides the foundation for evolving the two logics (independence and distinction, affirmation and negation). On this basis, it is only by extending C_3 to D_3 that the two specific logics are evolved.

We see that when using C_3 and FA to explain formal logic, we say that in C_3 it is 'A is A', and then in FA it is 'A is not non-A'. This, in fact, does not destroy the identity of A; it is still expressed from the perspective of A. This is a characteristic of formal logic. Let us express this from a different perspective. Similarly, we say that in C_3 it is 'A is A', and then in FA it is 'non-A is not A'. This is a symmetrical expression. However, this expression negates A and thus destroys the identity of A. When identity is destroyed, it must be re-established on a higher level, and the re-establishment of such an identity is its unification in D_3. This is a characteristic of dialectical logic. The characteristics of these two logics have already been discussed in the previous subsection "Dialectical Logic".

Through Z_2, then to No Form V, then to C_3, and then to D_3, we have evolved the mathematical structure of dialectical logic and formal logic. They are all constructed on such a mathematical model developed from the most basic Z_2, and are closely connected. This mathematical model shows that no form action theory is not only a philosophical framework, but also a systematic theory that can be rigorously deduced through mathematics. Through the mathematical evolution from Z_2 to D_3, no form action theory has transcended mere philosophical speculation to become a dynamic theoretical framework that can precisely describe logic. This theoretical model has leaped from a purely philosophical speculation to a method with the potential to generate new insights and testable predictions. This theoretical model is not only a philosophical breakthrough, but also a completely new method for constructing mathematics and logical science. This mathematical structure established for philosophy brings about an unfold-manifestation of how complexity can arise from the interaction of the simplest components.

We can combine the field VFc and No Form V × V. Since No Form V × V can be extended into a ring, and this ring is isomorphic to $F_4 \times F_4$, it is also isomorphic to VFc × VFc. We call this ring extended from No Form V × V VVRc (where R refers to a ring) = VFc × VFc. Thus, the additive operation of VVRc is $(x, y) + (m, n) = (x + m, y + n)$, and the multiplicative operation is $(x, y) \cdot (m, n) = (x \cdot m, y \cdot n)$, where the component-wise operations in addition and multiplication come respectively from the additive and multiplicative operations of VFc. (e, e) is the additive identity element of VVRc, and (c, c) is the multiplicative identity element of VVRc.

In this way, we can use the operations of both VFc and No Form V × V simultaneously in VVRc, thus combining them. For example, the addition of identity (a, c) and distinction (c, a) is being-for-itself (b, b), while their multiplication is the self (a, a). Let us attempt a philosophical interpretation: the multiplication of identity (a, c) and distinction (c, a) is equivalent to ignoring the isolation form in identity (a, c) and distinction (c, a) (because the multiplication of the components a and c is equivalent to ignoring the isolation c), and thus it becomes the self. From this perspective, the self is identity and distinction with the isolation form ignored: a self is itself, but it must also distinguish itself from itself.

(4) Iteration of the group

In fact, the dihedral group D_2 also has a reflection like the 'f' in D_3. Let us examine No Form V = {e, a, b, c}, which is isomorphic to D_2. Since any one of a, b, or c can serve as the reflection f, we can arbitrarily choose 'b' as f. Thus, for No Form V, we have C_2 = {e, a} and FA = {b, c} (where c = a · b). In this way, through the reflection action of 'b', C_2 (which is Z_2) and FA can be mutually transformed. Moreover, since any of a, b, or c can serve as the reflection f, we can choose the one we need as 'f' according to practical requirements. This reflection action in No Form V is a very useful method for finding on which concept two opposites are unified.

Below, we will use this method to find on which concept identity (a,c) and distinction (c,a) are unified:

Since identity (a,c), being-for-itself (b,b), and distinction (c,a) can constitute a no form integrated transformation, V_1 = {(e,e), (a,c), (b,b), (c,a)} is isomorphic to No Form V (that is to say, V_1 is a Klein four-group). Since (a,c) is viewing manifestation action from the perspective of isolation action, (a,c) is dominated by manifestation action. And since (c,a) is viewing isolation action from the perspective of manifestation action, (c,a) is dominated by isolation action. And (e,e) can be seen as pure no form. Therefore, (e,e) can be seen as 'e', (a,c) can be seen as 'a', (b,b) can be seen as 'b', and (c,a) can be seen as 'c'. We choose (b,b) as the reflection 'f'. Thus, in the direct product $V_1 \times V_1$, which is {(e,e), (a,c), (b,b), (c,a)} × {(e,e), (a,c), (b,b), (c,a)}, 'being (e,c)' becomes ((e,e), (c,a)).

The steps above can be further iterated. We find that ground, which was initially (e,c) (that is, being, whose ground is itself), will then become (((e,e),(e,e),...),((c,a),(a,c),...)), which can be simply denoted as: ([e], [c,a]). This represents the concrete ground. We see that initially, ground only contained the abstract isolation action 'c', but gradually it comes to contain identity (a,c) and distinction (c,a), as well as their more complex composite structures. In other words, identity (a,c) and distinction (c,a), as two opposing sides, are unified on ground. In this way, we have elucidated the dialectical unity of identity (a,c) and distinction (c,a).

This is a dialectical unity obtained through mathematical means, and it is consistent with the explanation obtained through conceptual analysis in the subsection "Dialectical Logic," which was that 'identity (a,c) and distinction (c,a) are unified on ground'. It is just that the dialectical unity obtained through mathematical means is clearer, more detailed, and more precise. This is the first time an instance of dialectical logic has been obtained through the method of mathematical operations. This is sufficient to show the reasonableness of no form action theory.

Thus, (e,c), as the most abstract ground, is in fact being. And concrete grounds all contain some level of identity and distinction. A concrete ground can be written as: ground([e], [c,a]). Therefore, such a limit expression for being, (e,c), is the most abstract ground. Such a mathematical deduction conforms to my previous analysis of the relationship between ground and being: starting from a thing, one continuously obtains higher-level grounds in the manner of a limit, and the final limit is being, thereby obtaining that being is the greatest ground, and that the ground of being is itself. For example, starting from a thing A, the ground of A is B, the ground of B is C..., and the final limit is being.

Similarly, transparency (e,a) is the most abstract opening; freedom (e,b) is the most abstract cause; manifestation (a,e) is the most abstract essence; motive force (b,e) is the most abstract subject; and isolation (c,e) is the most abstract substance.

Similarly, generation (b,c), the self (a,a), and independence (c,b) can constitute a no form integrated transformation. Freedom (e,b) thus becomes (((e,e),(e,e),…),((b,c),(c,b),…)). Through continuous iteration, we obtain that: generation (b,c) and independence (c,b) are unified on cause. A concrete cause can be written as: cause([e], [b,c]).

Similarly, change (b,a), self-limitation (c,c), and Immediacy (a,b) can constitute a no form integrated transformation. Transparency (e,a) thus becomes (((e,e),(e,e),…),((a,b),(b,a),…)). Through continuous iteration, we obtain that: change (b,a) and Immediacy (a,b) are unified on opening. A concrete opening can be written as: opening([e], [a,b]).

Since manifestation (a,e) and transparency (e,a) are in opposite directions, manifestation (a,e) thus becomes (((a,b),(b,a),…),((e,e),(e,e),…)). Therefore, change (b,a) and Immediacy (a,b) are unified on essence in an opposite manner. A concrete essence can be written as: essence([a,b], [e]).

Similarly, generation (b,c) and independence (c,b) are unified on the subject in an opposite manner. A concrete subject can be written as: subject([b,c], [e]).

Similarly, identity (a,c) and distinction (c,a) are unified on substance in an opposite manner. A concrete substance can be written as: substance([c,a], [e]).

Not only that, but we can also, based on V_1 = {(e,e), (a,c), (b,b), (c,a)}, iterate (c,a) into independence (c,b). Through continuous iteration, this becomes ([c,a], [b]) (referencing the earlier point: identity (a,c) and distinction (c,a) are unified on ground). Abbreviated, this is: iterating [c,a] into independence (c,b) turns it into ([c,a], [b]).

However, there is a slight difference here, which is that in this combination, there is the motive force 'b'. Since independence (c,b) is viewing isolation action from the perspective of motive force action, the question becomes one of viewing the isolation action in identity (a,c) and the isolation action in distinction (c,a) from the perspective of motive force action. Then, viewing the isolation action in identity (a,c) from the perspective of motive force action is affirmation (that is, 'A is B'). Viewing the isolation action in distinction (c,a) from the perspective of motive force action is negation. And affirmation and negation, as two opposing sides, are unified in independence. Through this combination, we can clearly see that both 'affirmation' and 'negation' contain the motive force 'b'. In other words, in the world of isolation, there are two motive forces: affirmation and negation.

Similarly, it can be obtained that:

Iterating [a,c] into Immediacy (a,b) turns it into ([a,c], [b]). From this, we obtain that: presence and absence, as two opposing sides, are unified in Immediacy (a,b). Note: ([a,c], [b]) and ([c,a], [b]) are different. ([a,c], [b]) is viewing manifestation from the perspective of motive force, while ([c,a], [b]) is viewing isolation from the perspective of motive force.

Iterating [b,c] into change (b,a) turns it into ([b,c], [a]). From this, we obtain that: appearance and disappearance, as two opposing sides, are unified in change (b,a).

Iterating [a,b] into identity (a,c) turns it into ([a,b], [c]). From this, we obtain that: direct and indirect, as two opposing sides, are unified in identity (a,c).

Iterating [c,b] into distinction (c,a) turns it into ([c,b], [a]). From this, we obtain that: homogeneity and difference, as two opposing sides, are unified in distinction (c,a).

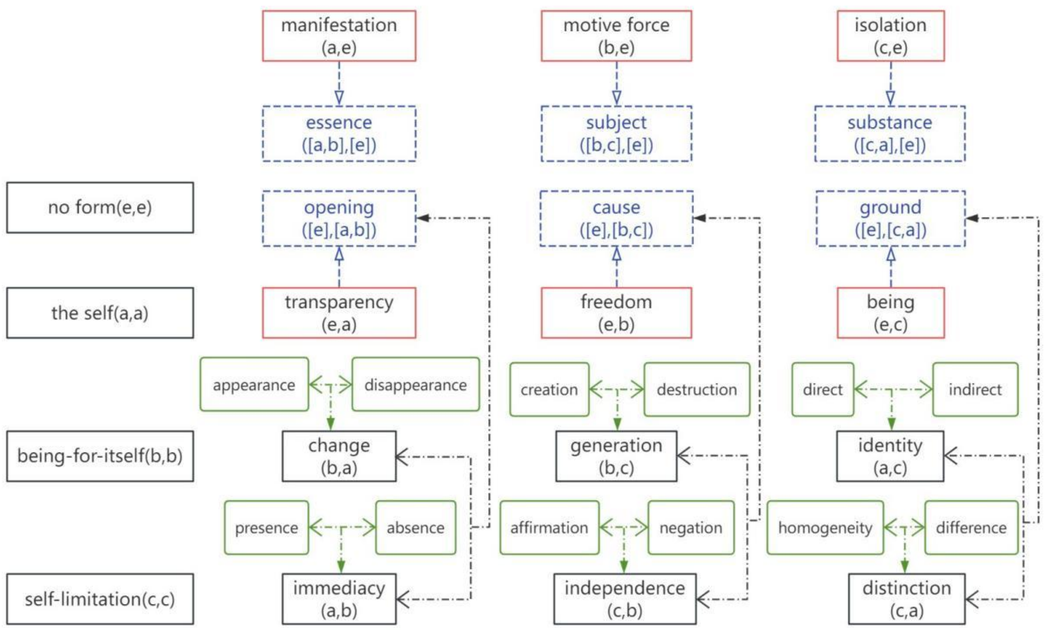
Iterating [b,a] into generation (b,c) turns it into ([b,a], [c]). From this, we obtain that: creation and destruction, as two opposing sides, are unified in generation (b,c).

Iterating [c,a] into independence (c,b) turns it into ([c,a], [b]). From this, we obtain that: affirmation and negation, as two opposing sides, are unified in independence (c,b). This has already been discussed.

We find that two opposing concepts can be iterated into different concepts in different ways and will produce different results of dialectical unity. [c,a] represents a dialectical unity composed of two opposing concepts, and it can exist in different concepts (unified objects).

Through iteration, we find that in these six groups, the original two-dimensional elements have become three-dimensional elements. For example, (c,b) has been turned into ([c,a], [b]). Through this iteration, non-trinitarian elements are turned into elements of the no form trinity.

These have already been explained through the method of conceptual analysis in the subsection "Dialectical Logic". However, now, through the method of mathematical operations, the opposition and unity among these concepts have become clearer and more precise. As shown in the diagram: (where the black and green arrows represent the unity of opposites among concepts, and the blue arrow represents the limit process.)



At this point, the core content of the framework of no form action theory that I have developed has all been constructed on a mathematical structure.

The iteration of the Klein four-group discussed above is a powerful method. As mentioned earlier, there are 35 Klein four-groups among the subgroups of No Form V × V. Iterating each of them will yield different results. Moreover, they can be cross-iterated (for instance, the first iteration uses {(e,e), (e,a), (b,b), (b,c)}, and the second iteration uses {(e,e), (e,a), (c,e), (c,a)}). This will yield extremely rich results. Through this method, we can precisely obtain the relationships among different concepts, and at the same time, we can also see how these concepts are precisely combined in a mathematical way.

(5) Argument on the Relationship Between Philosophy and Mathematics

The continuous extension starting from Z_2 (as no form and form) brings about an unfold-manifestation of the process of no form continuously transforming into form. Even if we do not first consider the mathematical structure within it, this transformation process must necessarily be a process of the continuous strengthening of form and the further clarification of structure. Therefore, this transformation process must necessarily be accompanied by a purely formal structure, and it must bring about an unfold-manifestation of itself through this structure. And since mathematics is the purest formal structure, this transformation process must necessarily possess such a pure, formal mathematical structure. Mathematical form is not merely a descriptive tool, but an ontological necessity for understanding and expressing the transformation from 'no form' to 'form'.

Our rational knowledge of this world is, in fact, the knowledge of isolation form, and the purest knowledge is mathematical knowledge. Therefore, the mathematization of philosophy is a necessity. But this is not to say that philosophy can be completely mathematized. In the previous subsection "The Relationship of Isolation: The no form action trinity," it was discussed that philosophy is a discipline that balances isolating thought (conceptual thought), motive force thought (logical, inferential thought), and manifesting thought (understanding thought).

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